

## ASYMMETRIC CONTROL CHARACTERISTICS OF DYE BDN ON THE TWO TRAVELLING WAVES IN A RING YAG LASER

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It is discovered experimentally that, when placing a plate of dye BDN in a ring YAG laser cavity, the dye shows asymmetric control actions on the clockwise and counter-clockwise travelling waves. Unidirectional dye *Q*-switched operation is realized in a ring cavity based on this property without the use of Faraday rotator, polarizer and half-wave plate. A simplified theoretical analysis of this phenomenon is presented.

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### I. ASYMMETRIC CONTROL CHARACTERISTICS

Figure 1 is a schematic diagram of a pulse ring YAG laser.  $M_1$  and  $M_3$  are total reflecting mirrors, while  $M_2$  is an output mirror with a reflectivity  $\beta$ ,  $D$  is a dye BDN *Q*-switch plate.

As the pulsed Xenon lamp flashes, the coherent emission of Nd:YAG will be built up along two ways. The clockwise branch, denoted by  $I'_+$ , will be attenuated by  $D$ , and reflected at  $M_2$ ,  $M_3$  and  $M_1$ , then enter into the working medium YAG where it is amplified. The other branch, the counter-clockwise branch,  $I'_-$ , travels along the opposite path. The dye plate will be bleached after many round-trips of the coherent light and thus a *Q*-switched giant pulse laser will be generated.

The experimental results are as follows:

1. If  $D$  is placed in any site between the left end of YAG rod and  $M_1$ ,  $M_1$  and  $M_3$  or  $M_3$  and  $M_2$ , it is found that

$$I'_{+,out} \gg I'_{-,out}.$$

2. If  $D$  is placed in any site between the right end of YAG rod and  $M_2$ , the result will be

$$I'_{-,out} \gg I'_{+,out}.$$

3. The above regularity will hold, irrespective of the change of  $\beta$ , the reflectivity of  $M_2$ . However, the ratio  $I'_{+,out}/I'_{-,out}$  (or  $I'_{-,out}/I'_{+,out}$ , in case 2) will increase with the decrease of  $\beta$ . It means that the unidirectionality becomes better.

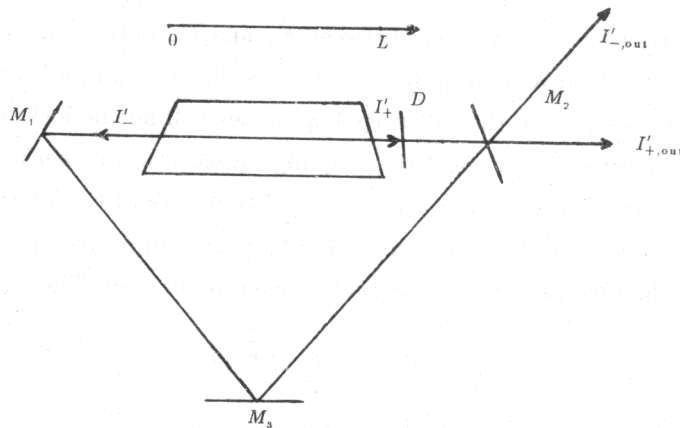


Fig. 1. Schematic diagram of a pulse ring YAG laser.

## II. THE MECHANISM OF ASYMMETRIC CONTROL

The dye BDN is a two-level saturable absorbing medium. However, as the  $Q$ -switched pulse occurs, the dye will be bleached and act in such a way just as a kind of optical nonlinear medium rather than an absorbing medium. By nonlinear interaction not only the phases between the two independent travelling wave modes of the ring cavity can be locked, but also the transfer from one mode to another and the quantitative energy transfer between two modes can be achieved naturally.

The interaction between the laser beam and the dye plate could be described by the degenerate four-wave mixing (DFWM) process, as shown in Fig. 2.

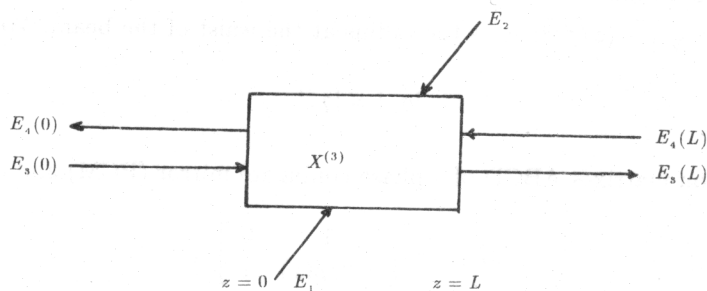


Fig. 2. Schematic diagram of DFWM.

Here the clockwise mode waves  $E_+$ ,  $E_-$  play the role of  $E_1$ ,  $E_2$  respectively and each of them may also play the role of signal in four-wave mixing to form a collinear DFWM process.<sup>[1,2]</sup> In Ref. [1] a four-wave mixing (FWM) phase-conjugate reflection coefficient 600%

for dye BDN has been reported. In Ref. [2], the phase conjugate reflection coefficient of CS<sub>2</sub> was measured by collinear DFWM and polarization separation techniques.

To discuss the transfer characteristics between  $E_+$  and  $E_-$  in the nonlinear action of the dye plate first, a semi-classical analysis of two-mode bi-directional ring laser is used, and it shows that<sup>[3]</sup> the frequencies of the two travelling modes will not be locked if the coupling between them is neglected. However, if the coupling caused from the back-scattering of the optical reflecting mirror is taken into account, the frequencies of the two travelling waves will be locked. Because of the existence of FWM phase conjugate back-scattering, the frequency-lock of the two travelling modes will happen in our case. Therefore,

$$\omega_+ = \omega_- , \quad k_+ = k_- , \quad (1)$$

and if a starting time is appropriately chosen, we will have

$$\varphi_+ = \varphi_- = 0 . \quad (2)$$

The complex amplitudes of the two modes can be expressed as<sup>[4,5]</sup>

$$E_+ = A_0 f |q_+|^{-1} \cdot \exp(-ik_+ r^2 / 2q_+) \cdot \exp[-i(\omega_+ t + k_+ z - \arctan z/f)] , \quad (3)$$

$$E_- = A_0 f |q_-|^{-1} \cdot \exp(-ik_- r^2 / 2q_-) \cdot \exp[-i(\omega_+ t - k_- z + \arctan z/f)] , \quad (4)$$

where  $q_+$  and  $q_-$  are the complex parameters of  $E_+$  and  $E_-$ , respectively, defined by

$$q_+^{-1} = \rho^{-1}(z) - i\lambda/\pi W^2 , \quad (5)$$

$$q_-^{-1} = \rho^{-1}(-z) - i\lambda/\pi W^2 = -\rho^{-1}(z) - i\lambda/\pi W^2 , \quad (6)$$

where  $\rho(z)$  is the radius of curvature of the wave front and  $W(z)$  is the radius of light spot at  $z$ ,  $W(z) = W_0 \sqrt{1 + (z/f)^2}$ ;  $\omega_0$  is the radius at the waist of the beam. We also have

$$|q_+| = |q_-| .$$

The reflection matrixes ABCD of a phase-conjugate mirror (PCM) and a flat mirror are

$$M_{\text{pcm}} = \begin{bmatrix} 1 & 0 \\ -2/\rho(z) & 1 \end{bmatrix} , \quad (7)$$

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} . \quad (8)$$

The transformation formula of the complex parameter  $q$  is

$$q_2 = (Aq_1 + B)/(Cq_1 + D) . \quad (9)$$

The PCM reflection transformation between  $E_+$  (or  $E_-$ ) and  $E_{+a}$  (or  $E_{-a}$ ) can be obtained as

$$E_{+a} = A_0 f |q_{+a}|^{-1} \cdot \exp(-ik_+ r^2 / 2q_{+a}) \cdot \exp[-i(\omega_+ t - k_+ z + \arctan z/f)] . \quad (10)$$

From Eqs. (9), (5) and (7), we have

$$q_{+a} = (Aq_+ + B)/(Cq_+ + D) = q_+ [-q_+(2/\rho(z)) + 1]^{-1}$$

or

$$q_{+a}^{-1} = 2\rho^{-1}(z) + q_+^{-1} = -\rho^{-1}(z) - i\lambda/\pi W^2 = q_-^{-1} . \quad (11)$$

Substituting Eqs. (11), (1) into Eq. (10) and taking Eq. (4) into account yield

$$E_{+a} = A_0 f |q_-|^{-1} \cdot \exp(-ik_- r^2 / 2q_-) \cdot \exp[-i(\omega_- t - k_- z + \arctan z/f)] = E_- . \quad (12)$$

By the same deduction we have

$$E_{-a} = E_+ . \quad (13)$$

By considering the back-scatter reflectivity of PCM, the partial transfer will happen between  $E_+$  and  $E_-$ .

It can be seen from Eqs. (12) and (13) that natural transfer between the two travelling ring-cavity modes can be achieved by FWM phase-conjugate back-scattering of BDN without occurrence of other modes. Based on this principle a single longitudinal operation of a ring-cavity  $Q$ -switched dye-laser has been realized experimentally at our laboratory.

If the PCM is replaced by a flat-reflection mirror, then from Eqs. (6), (8) we have

$$q_{+a} = (Aq_+ + B)/(Cq_+ + D) = q_+ ,$$

i. e.

$$q_{+a}^{-1} = q_+^{-1} . \quad (14)$$

Substituting Eq. (14) into Eq. (10), together with Eq. (1), gives

$$E_{+a} = A_0 f |q_+|^{-1} \cdot \exp(-ik_- r^2 / 2q_+) \cdot \exp[-i(\omega_- t - k_- z + \arctan z/f)] \neq E_- \quad (15)$$

and

$$E_{-a} \neq E_+ . \quad (16)$$

The difference between  $E_{+a}(E_{-a})$  and  $E_-(E_+)$  lies mainly in the difference of radius of curvature  $\rho(z)$  of the wave front. In general, a mode interference will occur.

Finally, let us examine the energy transfer between the two travelling modes on the basis described above and consider the action of the gain medium. The analysis is similar to that reported in Refs. [6, 7].

Introducing normalized light intensity ( $|E|^3 \sim I$ ) yields

$$I_+ = I'_+/I'_s, I_- = I'_-/I'_s, \quad (17)$$

where  $I'_s$  is the saturation light intensity of the gain medium. The gain coefficient  $G$  is defined as

$$G = I_+^{-1} dI_+/dz = -I_-^{-1} dI_-/dz. \quad (18)$$

The saturation gain of homogeneously broadened gain medium Nd:YAG can be abbreviated as

$$G = G_0/[1 + (I_+ + I_-)], \quad (19)$$

where  $G_0$  is the small signal gain. By integrating Eq. (18) we get

$$I_+(z)I_-(z) = I_+(0)I_-(0) = I_+(L)I_-(L) = 0. \quad (20)$$

To examine the light intensity variation outside the gain medium, i.e., at  $z = 0, L$ , a phase conjugate back-scatter reflectivity  $R$  and a forward transmittance  $T$  of the dye plate are introduced phenomenologically.

$$I_+(0) = T\beta I_+(L) + R\beta^2 I_-(0), \quad (21)$$

$$I_-(L) = T\beta I_-(0) + R I_+(L). \quad (22)$$

As the  $Q$ -switched laser output is formed after the dye has been bleached, the absorption loss of light by dye-plate is very small and can be neglected. Because of the energy conservation, using Eqs. (21) and (22), we have

$$I_+(L) + \beta I_-(0) = T I_+(L) + R I_+(L) + T\beta I_-(0) + R\beta I_-(0) = I_+(0)/\beta + I_-(L). \quad (23)$$

Substituting Eq. (20) into Eq. (23) gives

$$\beta I_-(0) I_+^2(L) + [\beta^2 I_-^2(0) - C] I_+(L) - \beta I_-(0) C = 0. \quad (24)$$

Equation (24) has two solutions

$$I_+(L) = \frac{[C - \beta^2 I_-^2(0)] \pm \sqrt{[C - \beta^2 I_-^2(0)] + 4\beta^2 I_-^2(0)C}}{2\beta I_-(0)}, \quad (25)$$

i.e.,

$$I_+^{(1)}(L) = C/\beta I_-(0), \quad I_+^{(2)}(L) = -\beta I_-(0). \quad (26)$$

Equation (26) gives

$$I_+(0) = C/I_-(0) = I_+(L)\beta, \quad I_-(L) = C/I_+(L) = I_-(0)\beta,$$

which is not consistent with the experimental results and should be rejected. The second solution gives a result  $I_+(L) = -\beta I_-(0)$ , where the minus sign means the opposite travelling directions of  $I_+(L)$  and  $I_-(0)$ , i.e.,

$$|I_-(0)|/|I_+(L)| = \beta^{-1}. \quad (27)$$

The solution we are interested in most is the ratio of the output light intensities of the two beams ( $I_+$  and  $I_-$ ), i.e., the unidirectionality. From Fig. 1, we get

$$I_{-,out} = (1 - \beta)I_-(0), \quad (28)$$

$$I_{+,out} = (1 - \beta)[TI_+(L) + R\beta I_-(0)] = (1 - \beta)[T\beta I_-(0) + \beta R I_-(0)]. \quad (29)$$

Therefore

$$I_{-,out}/I_{+,out} = 1/\beta(T + R). \quad (30)$$

The condition  $R + T = 1$  should hold according to the energy conservation relation. Thus  $I_{D,in}$ , the intensity of the incident light on  $D$ , and  $I_{D,out}$ , the intensity of light leaving  $D$  can be written as

$$I_{D,in} = I_+(L) + \beta I_-(0),$$

$$I_{D,out} = RI_+(L) + (1 - R)I_+(L) = \beta RI_-(0) + \beta(1 - R)I_-(0) = I_+(L) + \beta I_-(0) = I_{D,in}.$$

On the other hand, if we substitute  $T + R = 1$  in Eq. (3), then we have

$$I'_{-,out}/I'_{+,out} = I_{-,out}/I_{+,out} = \beta^{-1}. \quad (31)$$

For the case of placing dye plate  $D$  at the left side of YAG rod we have

$$I'_{+,out}/I'_{-,out} = I_{+,out}/I_{-,out} = \beta^{-1}. \quad (32)$$

Thus, the ratio  $I'_{-,out}/I'_{+,out}$  is inversely proportional to  $\beta$  under the condition of energy conservation. It is interesting to note that, when only the energy conversion is concerned, a dye plate is equivalent to a linear optical element with reflectance  $R$  and transmittance  $T$ .

However, there is a basic difference between a dye plate and an optical mirror. A PCM has the wave front inversion property. Thus it does not matter where the PCM is placed in the cavity and how the state of PCM is adjusted. The conversion between the two travelling-wave modes can always be achieved naturally without the occurrence of other mode. However the mode-conversion by a linear plane mirror can be achieved only when mirror is placed at the beam waist and a strict adjustment is needed to prohibit occurrence of other modes.

Table 1 lists the results obtained using two output mirrors whose reflectivities are  $\beta = 13.2\%$ , and  $4\%$ , respectively.  $W_{\pm i} = I_{\pm i}\tau_i$ ,  $W_{\pm i}$  is the single pulse output energy and  $\tau_i$  is the pulse duration width. Results of measurements also are given in Table 1.

Table 1.

Reflectivity of output mirror $\beta/\%$	Experimental $\langle W_{-i} \rangle / \langle W_{+i} \rangle$	Calculation $\beta^{-1}$
13.2	7.2	7.6
4	21	25

The calculations agree well with the experimental results.

Our conclusion is that the unidirectional operation of a ring dye laser is essentially determined by the reflectivity  $\beta$  of the output mirror. Based on the above conclusion a dye Q-switched, single longitudinal mode, ring Nd:YAG laser has been designed and carried out successfully.

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